DESIGN OF A CONTROLLER ALLOWED THE INTUITIVE CONTROL OF AN X4-FLYER.

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Abstract: In this paper, we present a control design for the tele-operation of a miniature unmanned aerial vehicle known as an X4-flyer. A simple dynamic nonlinear model for the vehicle, valid for quasi-stationary flight conditions, is derived as a basis for the control design. To ensure the vehicle remains in the quasi-stationary flight regime the controller incorporates barrier function on the closed-loop system velocity. The proposed control design, based on control Lyapunov function analysis, is simple, robust and can be easily implemented on board the experimental vehicle. Simulations and partial experimental results are provided that validate the performance of the closed-loop system.

Keywords: Rotary Wing, Unmanned aerial vehicle, X4-flyer, Nonlinear control, Lyapunov analysis, Under-actuated system.

1. INTRODUCTION

The interest for the remote control hovering system increases more and more particulary in the military purposed for the surveillance and inspection in dangerous or awkward environments. For example, one can imagine the use of such vehicles in order to explore a contaminated area before a human intervention. In this paper, we are interested in the remote control problem of an unmanned aerial vehicle known as the "X4-flyer" which is a four rotors vertical take off and landing (VTOL) vehicle capable to perform stationary or quasi stationary flights. The mechanical simplicity of the "X4-flyer" allows to have a reliable and commercially exploitable system. However, contrary to a remote-controlled car, the manual control of a flying system such as an X4-flyer with a joystick needs several hours of training before being able to succeed (Young and Aiken, 2002). Consequently, we are interested in the assisted

Fig. 1. The X4-flyer

manual control of the X4-flyer in order to allow a neophyte to control it without difficulty. The idea is to control the vehicle in sending simple orders (go on the left, forward,...). Although recent studies about the teleoperation of hovering system have been started, the teleoperation of an unmanned aerial vehicle has, in our knowledge, never been experimented on a real "X4-flyer".

In this paper, we are particulary interested in the assisted manual control which will allow to a neophyte to control this vehicle without difficulty. The idea consists in elaborating an embedded controller which allows, from information issued from a simple joystick and measurements of an Inertial Measurement Unit (IMU), the stabilization of the translational velocity and the yaw angle of the X4-flyer. More precisely, we propose in this paper to elaborate a simple nonlinear control law of an "X4-flyer" unmanned aerial vehicle (cf. figure 1) insuring quasi-stationary flight by bounding its translational speed and the vehicle orientation in order to limit the attitude in a a small neighborhood around the inertial gravity direction. In contrast to classical remote-control orders consisting in sending forces and torques to the vehicle, the proposed approach uses intuitive orders such translational velocities and yaw angle as desired inputs sent from the ground to the aerial vehicle.

The paper is arranged into five sections. In the second section a dynamic modeling of the "X4-flyer" is presented. in the third section a non linear control is designed based on Lyapunov function analysis. In section 4, partial experimental results are presented. Finally, section 5 presents some concluding remarks.

2. DYNAMIC MODELING

The X4-flyer is an omnidirectional VTOL (vertical take off and landing) vehicle ideally suited for stationary and quasi-stationary flight conditions. The control of an X4-flyer is achieved by the differential control of the thrust generated by each propeller. Up/down motion is controlled by collectively increasing or decreasing the thrust of all four motors. The thrust difference between the forward and the rear rotor (resp. the left/right rotor) creates a pitch torque inducing translation forward/rear (resp. left/right) motion. To finish, let's see the yaw control. When a propeller turns, it has to overcome air resistance. In the X4-flyer, both sets of front-rear and left-right motors turn in opposite directions (cf Figure 2). Moreover, the reactive torque is essentially a function of the propeller rotational velocity. Consequently, controlling the X4-flyer yaw is equivalent to control the sum of reactive torques. As long as all rotors produce the same reactive torque (all rotors turn at the same speed), the sum of all reactive torques is zero and there is no yaw motion. The X4-flyer modeling is inspired from (Hamel *et al.*, 2002). In order to model the system dynamics, we define two frames shown in figure 2,

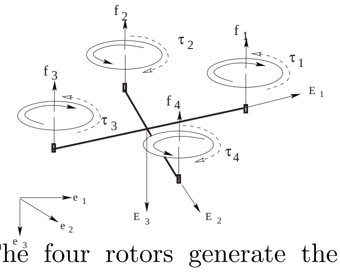

Fig. 2. The four rotors generate the collective thrust. For each propeller, the rotation direction is represented in solid line and the reactive torque in dot line.

- an inertial frame $\mathcal{R}_i$ defined by (e_1, e_2, e_3) which is attached to the earth, relative to a fixed origin. It is assumed to be Galilean.
- a body fixed frame $\mathcal{R}_a$ defined by (E_1, E_2, E_3) which is attached to the center of mass of the vehicle.

The position of the center of mass of the vehicle with respect to the inertial frame $\mathcal{R}_i$ is denoted x. Let v (resp. Ω) denotes the linear (resp. angular) velocity of the center of mass expressed in the inertial frame $\mathcal{R}_i$ (resp. body fixed frame $\mathcal{R}_a$). Define the attitude R of the body fixed frame with respect to the inertial frame by means of Euler angles ξ =(yaw(ϕ),pitch (θ) and roll (ψ)).

$$R = \begin{pmatrix} c_\theta c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\psi s_\theta s_\phi - s_\psi c_\phi \\ -s_\theta & s_\psi c_\theta & c_\psi c_\theta \end{pmatrix} \quad (1)$$

where the following shorthand notations for trigonometric functions are used: c_α=cos(α), s_α=sin(α), t_α=tan(α). According to (Murray *et al.*, 1994), the evolution of R is given by

$$\dot{R} = R\text{sk}(\Omega) \quad (2)$$

where sk(Ω) is an antisymmetric matrix such that sk(Ω)v is the vector cross product between Ω and v. Let $\Gamma \in \mathcal{R}_a$ be the control torques derived from differential thrusts provided by the rotors on the airframe in order to modify the attitude of the vehicle. Let T be the global thrust generated by the propellers rotation. Newton's equations of motion yield

$$\begin{aligned} m\dot{v} &= -TRe_3 + mge_3 \\ \mathbf{I}\dot{\Omega} &= -\Omega \times \mathbf{I}\Omega + \Gamma. \end{aligned} \quad (3)$$

where m represents the mass of the airframe, $\mathbf{I}$ its inertia matrix expressed in $\mathcal{R}_a$ supposed diagonal and g the gravity. The lift f_i generated by the rotor i turning at the speed ω_i in free air may be expressed as (Prouty, 1995)

$$f_i = -b\omega_i^2 E_3 \quad (4)$$

where b is a positive constant depending on air density, rotor blades collective pitch and geometric blade characteristics. The reactive torque due to the rotor drag generated by a rotor in free air may be modeled as

$$\tau_i = -\kappa\omega_i|\omega_i|E_3 \quad (5)$$

Where the positive constant κ depends on air density, rotor blades collective pitch and geometric blade characteristics. With the above consideration and considering that propellers have a symmetrical disposition around the gravity center G with d as offset of each propeller from the center of mass, we can deduce the expression of the control torques Γ and the expression of the global thrust T generated by the rotation of the four motors:

$$\begin{pmatrix} T \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{pmatrix} = \begin{pmatrix} b & b & b & b \\ 0 & db & 0 & -db \\ db & 0 & -db & 0 \\ -\kappa & \kappa & -\kappa & \kappa \end{pmatrix} \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} = A \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} \quad (6)$$

Due to the flexibility of the propellers, gyroscopic effects generated by the propellers rotation are not transmitted to the body frame but the generated blink of the fans modify a little the rotation plan of the propellers. As both sets of front-rear and left-right motors turn in opposite directions and due to the symmetry of the body frame, these effects can be considered as negligible.

Consequently, the dynamic model of the "X4-flyer" is

$$\dot{x} = v \quad (7)$$

$$\dot{v} = ge_3 - \frac{1}{m}TRe_3 \quad (8)$$

$$\dot{R} = R\mathrm{sk}(\Omega) \quad (9)$$

$$\mathbf{I}\dot{\Omega} = -\Omega \times \mathbf{I}\Omega + \Gamma \quad (10)$$

and T and Γ can be deduced from ω via (6).

3. CONTROL LAW

In this section, we propose to develop a nonlinear controller for the above dynamics based on saturating the linear dynamics for bounding the vehicle orientation and limiting it to very small values. The control strategy is based on Lyapunov functions. The control problem considered consists in finding a control action $\varpi = (\omega_1^2...\omega_4^2)$ from equation (6) depending only on the state measurements (v, R, Ω) and parameters of the desired trajectory (v_d, ϕ_d). In order to pilot the X4-flyer, we will separate the control design into two parts: translational and rotational dynamics control design.

- For the translational dynamics the thrust will be assigned and the full desired orientation will be defined.
- For the rotational dynamics the control torques are assigned.

From the assigned thrust and torques, the desired orientation velocity of each propeller (ω_i) is obtained using equation (6). Each propeller speed will then be controlled with fast transient to obtain the desired orientation speed (ω_i).

3.1 Stabilization of the translational dynamics

In this section, we propose a nonlinear controller allowing the convergence of the translation velocity and yaw angle to a desired linear velocity and a desired yaw respectively. Moreover, in order to insure that the modeling stays available, we have to insure that the vehicle remains in quasi-stationary flight conditions that we define as follows:

- v, the linear velocity, is moderated ($|v| < v_{\lim}$),
- T, the global thrust, verifies the following inequality:

$$mg - \delta \leq T \leq mg + \delta, \qquad \text{for } 0 < \delta << mg$$

In order to satisfy these requirements, the following natural assumptions are required:

$$|v_d| < v_{\lim}, \qquad \text{and} \qquad |\dot{v}_d| < \beta << mg$$

Define the error

$$\varepsilon_1 = v - v_d \quad (11)$$

Let E_T be the first storage function for the translational dynamics, defined as follows:

$$E_T = \frac{1}{2}m|\varepsilon_1|^2 \quad (12)$$

Deriving E_T, it yields:

$$\begin{aligned} \dot{E}_T &= m\varepsilon_1^T(\dot{v} - \dot{v}_d) \\ &= \varepsilon_1^T(-TR^d e_3 + mge_3 - m\dot{v}^d - TR(I_3 - \tilde{R}))e_3 \end{aligned}$$

where I_3 is the identity matrix and $\tilde{R}$ is the attitude deviation ($\tilde{R} = R^T R^d$). The term $R^d e_3$ represents the desired orientation of the global thrust T. The attitude will be designed to tilt the airframe of the X4-flyer in order to track this orientation while insuring the convergence of the yaw angle around this Re_3 axis to the desired one. It exist a number of possibilities for fully determining the full desired orientation R^d. The key point is that R^d is fully defined by the vectorial constraint on $R^d e_3$ combined with the desired yaw. Note that, as the system is under-actuated, the translational dynamics is controlled by the term $TR^d e_3$. Consequently, the translational control is perturbed by the rotational dynamics. More precisely, as the term TR_3^d is considered as control input vector for the translational dynamics, the term $(I_3 - \tilde{R})$ will be considered as disturbance and corresponds to the error term to stabilize for the orientation dynamics (see section 3.2). Assigning $TR^d e_3$ immediately from the storage function E_T and then controlling the attitude dynamics leads to time scale separation between the attitude and linear dynamics.

In order to preserve the quasi-stationary conditions presented above, we suggest to limit the energy E_T to a known and fixed bound noted α. Several technics may be considered. In this paper we propose the use of barrier function (previously proposed in ((Ngo *et al.*, 2004)) using the following reformulation of the storage function E_T:

$$E_T^n = \ln(\frac{\alpha}{\alpha - E_T}) \tag{13}$$

Note that E_T^n can be considered as a Lyapunov function candidate for the translational dynamics because if $E_T = 0$, $E_T^n(0) = 0$ and if $E_T \to \alpha$ $E_T^n \to \infty$.

Lemme 1. Let α, β, δ and ϵ be four positive constants such that

$$\beta < \frac{\delta}{m}, \ \epsilon << \alpha$$

Assume that:

$$|v_d| < \frac{1}{2}\sqrt{\frac{\alpha}{m}}, \quad \text{and} \quad |\dot{v}_d| < \beta.$$

Assume that exists a controller for the orientation dynamics insuring exponential convergence of the error term $(I_3 - \tilde{R})$, let γ and τ be two positive constants such that, for $\tau > \frac{\gamma(mg+\delta)}{\sqrt{2m}(\sqrt{\alpha}-\sqrt{\alpha-\epsilon})}$

$$||I_3 - \tilde{R}||_F < \gamma \exp(-\tau t)$$

where $||.||_F$ represents the Frobenius norm. Consider the following control for the translational dynamics:

$$TR^d e_3 = -m\dot{v}^d + mge_3 + k_1(\alpha - E_T)\varepsilon_1, \tag{14}$$

if $k_1 < \frac{\sqrt{m}(\delta - m\beta)}{\sqrt{2}\alpha^{\frac{3}{2}}}$, then for all initial condition for translational velocity $|v(0)| < \frac{1}{2}\sqrt{\frac{\alpha}{m}}$ such that $E_T(0) < \alpha - \epsilon$, it follows that $\forall t > 0$:

$$E_T(t) < \alpha \tag{15}$$

$$mg - \delta < T < mg + \delta \tag{16}$$

$$E_T^n \text{ converges asymptotically to zero} \tag{17}$$

$$|v(t)| < (\sqrt{2} + \frac{1}{2})\sqrt{\frac{\alpha}{m}} \tag{18}$$

Proof: First we have to prove that the proposed control law (14) stabilize the translational dynamics. Tacking the time derivative of (13), it follows

$$\dot{E}_T^n = \frac{\dot{E}_T}{\alpha - E_T} \tag{19}$$

Substituting $\dot{E}_T$ by its expression, it yields

$$\dot{E}_T^n = \frac{\varepsilon_1^T(-TR^d e_3 + mge_3 - m\dot{v}^d - TR(I_3 - \tilde{R}))e_3}{\alpha - E_T} \tag{20}$$

Introducing now the expression of the desired control $TR^d e_3$ (14), it yields

$$\dot{E}_T^n = -k_1|\varepsilon_1|^2 - \frac{1}{\alpha - E_T}\varepsilon_1^T TR(I_3 - \tilde{R})e_3 \tag{21}$$

Thus

$$\dot{E}_T = -k_1(\alpha - E_T)|\varepsilon_1|^2 - \underbrace{\varepsilon_1^T TR(I_3 - \tilde{R})e_3}_{\text{rotational dynamics}} \tag{22}$$

If $E_T < \alpha$, $\forall t > 0$, then, from the second assumption of lemma 1, one can insure the asymptotic convergence of the storage function E_T subject to exponentially vanishing perturbation (see (Khalil, 1992)).

It remains to show that 15-16 are valid $\forall t > 0$. At time zero, bounds 15-16 are satisfied directly by the lemma statement. Moreover, the evolution of the state of the "X4-flyer" is smooth and we may proceed using a proof by contradiction. Assume that the bounds 15-16 are not valid for all time. Then there exist a first time t_0 such that for all $t \in [0, t_0[$ the bounds 15-16 are valid. The contradiction is shown case by case. Assume that $E_T(t_0) = \alpha$ and that $mg - \delta \leq T \leq mg + \delta$ on the interval $t \in [0, t_0]$. Let us prove that the bound E_T cannot be the first bound which is broken. Let $E_T(0) < \alpha - \epsilon$ and assume the validity of (16), it yields

$$\dot{E}_T \leq -k_1\varepsilon_1^T\varepsilon_1(\alpha - E_T) + |\varepsilon_1|(mg + \delta)||I_3 - \tilde{R}||_F \tag{23}$$

Knowing that

$$||I_3 - \tilde{R}||_F < \gamma \exp(-\tau t),$$

it follows:

$$\dot{E}_T \leq \gamma|\varepsilon_1|(mg + \delta)\exp(-\tau t) \tag{24}$$

Knowing that $E_T = \frac{1}{2}m|\varepsilon_1|^2$, it follows that:

$$|\dot{\varepsilon}_1| < \frac{\gamma}{m}(mg + \delta)\exp(-\tau t) \tag{25}$$

By integration, it yields

$$|\varepsilon_1(t)| < \sqrt{\frac{2(\alpha - \epsilon)}{m}} + \frac{\gamma(mg + \delta)}{m\tau} \tag{26}$$

Consequently, tacking τ such that

$$\tau > \frac{\gamma(mg + \delta)}{\sqrt{2m}(\sqrt{\alpha} - \sqrt{\alpha - \epsilon})} \tag{27}$$

yields $E_T < \alpha$. This ensures that E_T cannot be the first bound which is broken. Assume that $T(t_0) = mg \pm \delta$, $E_T(t_0) < \alpha$ and proof that T cannot be broken. Knowing that $T = |-m\dot{v}^d + mge_3 + k_1\varepsilon_1(\alpha - E_T)|$ using the following bounds:

$$E_T < \alpha, \quad |\dot{v}_d| < \beta \quad \text{and} \quad (\alpha - E_T)\varepsilon_1 \leq \alpha\sqrt{\frac{2\alpha}{m}} \tag{28}$$

it yields

$$mg - k_1\alpha\sqrt{\frac{2\alpha}{m}} - m\beta < T < mg + k_1\alpha\sqrt{\frac{2\alpha}{m}} + m\beta \tag{29}$$

If $k_1 < \frac{\sqrt{m}(\delta - m\beta)}{\sqrt{2}\alpha^{\frac{3}{2}}}$, we deduce that relation (16) cannot be the first bound which is broken. Let us prove that the linear velocity is bounded according to (18). Knowing that $E_T < \alpha$ is valid $\forall t$ it yields

$$-\sqrt{\frac{2\alpha}{m}} < (v - v_d) < \sqrt{\frac{2\alpha}{m}} \tag{30}$$

Consequently, knowing that $|v_d| \geq \sqrt{\frac{\alpha}{m}}$, it follows

$$|v(t)| < (\sqrt{2} + \frac{1}{2})\sqrt{\frac{\alpha}{m}} \tag{31}$$

△

3.2 Control of the rotational dynamics

The next stage of the control design involves the control of the attitude dynamics such that the error $(\tilde{R} - I_3)$ converges exponentially to zero. Designing a controller to stabilize the above term is a difficult problem. Note however that this can be done using the quaternions representation of $\tilde{R}$ (N. Guenard and V.Moreau, 2005), Frobenius or directly the log of the error term (Murray *et al.*, 1994). In this paper, we propose an elegant controller for the orientation dynamics based on the Frobenius norm of the error term.

$$||I_3 - \tilde{R}||_F = \sqrt{\mathrm{tr}\left((I_3 - \tilde{R})^T(I_3 - \tilde{R})\right)} \tag{32}$$

Introducing at this step of the study the backstepping procedure, let ε_R be the error function such that $\varepsilon_R = I_3 - \tilde{R}$. Developing (32), it yields:

$$||\varepsilon_R||_F = \sqrt{2\mathrm{tr}(I_3 - \tilde{R})} \tag{33}$$

Let define the first storage function E_R as

$$E_R = \frac{1}{4}||\varepsilon_R||_F^2 = \frac{1}{2}\mathrm{tr}(I_3 - \tilde{R}) \tag{34}$$

Tacking the time derivative of (34), it yields

$$\begin{aligned}\mathrm{tr}(\dot{\tilde{R}}) &= \mathrm{tr}\left(-\mathrm{sk}(\Omega)\tilde{R} + \tilde{R}\mathrm{sk}(\Omega_d)\right) \\ &= \mathrm{tr}\left(-\mathrm{sk}(\Omega)\tilde{R} + \mathrm{sk}(\Omega_d)\tilde{R} + \tilde{R}\mathrm{sk}(\Omega_d) - \mathrm{sk}(\Omega_d)\tilde{R}\right)\end{aligned} \tag{35}$$

Note that

$$\tilde{R}\mathrm{sk}(\Omega_d) - \mathrm{sk}(\Omega_d)\tilde{R} = [\tilde{R}, sk(\Omega_d)]$$

is a Lie bracket. Using the linearity of "sk" operator and that $\mathrm{tr}[\tilde{R}, sk(\Omega_d)] = 0$, (35) becomes

$$\dot{E}_R = \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\Omega - \Omega_d)\tilde{R}\right) \tag{36}$$

Decomposing $\tilde{R}$ in a sum of symmetrical and antisymmetrical matrix noted respectively $\pi_s\tilde{R}$ and $\pi_a\tilde{R}$ such that

$$\pi_s\tilde{R} = \frac{\tilde{R} + \tilde{R}^T}{2}, \qquad \pi_a\tilde{R} = \frac{\tilde{R} - \tilde{R}^T}{2} \tag{37}$$

it yields

$$\dot{E}_R = \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\Omega - \Omega_d)\pi_s\tilde{R}\right) + \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\Omega - \Omega_d)\pi_a\tilde{R}\right) \tag{38}$$

Moreover, we can easily demonstrate that the trace of a symmetric and an antisymmetric matrix product is equal to zero and as "sk" is an antisymmetrical matrix, (38) becomes:

$$\dot{E}_R = \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\Omega - \Omega_d)\pi_a\tilde{R}\right) \tag{39}$$

For the second step of the study, we consider as virtual control input Ω (Ω^v) such that, if Ω^v is considered as control input, the following choice insures the convergence of E_R to zero.

$$\Omega^v = \Omega_d - k_2\mathrm{vex}\left(\pi_a\tilde{R}^T\right) \tag{40}$$

where $k_2 > 0$ is a positive gain and "vex" represents the inverse operator of "sk". The purpose now is to insure the convergence of the error term $(\Omega - \Omega^v)$ to zero. Define the error term δ as

$$\delta = \Omega - \Omega^v \tag{41}$$

In order to stabilise δ to zero, let define a new storage function defined as follows:

$$W = \frac{1}{2}\mathrm{tr}(I_3 - \tilde{R}) + \frac{1}{2}|\delta|^2 \tag{42}$$

Tacking the time derivative of (42),

$$\dot{W} = \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\Omega^v - \Omega_d + \delta)\pi_a\tilde{R}\right) + \delta^T\dot{\delta} \tag{43}$$

Deriving δ and substituting Ω^v by its expression, it comes

$$\begin{aligned}\dot{W} = &-\frac{1}{2}k_2||\pi_a\tilde{R}||_F^2 + \frac{1}{2}\mathrm{tr}\left(\mathrm{sk}(\delta)\pi_a\tilde{R}\right) \\ &+ \delta^T\left[\dot{\Omega} - \dot{\Omega}_d + k_2\frac{d}{dt}\left(\mathrm{vex}(\pi_a\tilde{R}^T)\right)\right]\end{aligned} \tag{44}$$

Remark 1. Let A and B two antisymmetrical matrix, we can easily show that $\mathrm{tr}(AB) = -2\mathrm{vex}(A)^T\mathrm{vex}(B)$.

Consequently, at the view of the above remark, (44) becomes

$$\begin{aligned}\dot{W} = &-\frac{1}{2}k_2||\pi_a\tilde{R}||_F^2 - \delta^T\mathrm{vex}(\pi_a\tilde{R}) \\ &+ \delta^T\left[\dot{\Omega} - \dot{\Omega}_d + k_2\frac{d}{dt}\left(\mathrm{vex}(\pi_a\tilde{R}^T)\right)\right]\end{aligned} \tag{45}$$

At this step of our study, note that the relation between $\dot{\Omega}$ and Γ is algebraic (Eq. 10). In order to control the rotational dynamics, we consider Γ as

$$\begin{aligned}\Gamma = &\mathbf{I}\left[\dot{\Omega}_d - k_2\frac{d}{dt}\left(\mathrm{vex}(\pi_a\tilde{R}^T)\right) + \mathrm{vex}(\pi_a\tilde{R}) - k_3\delta\right] \\ &+ \Omega \times I\Omega\end{aligned} \tag{46}$$

with $k_3 \geq 0$. Using the above controller in time derivative of the storage function, it yields

$$\dot{W} = -\frac{1}{2}k_2||\pi_a\tilde{R}||_F^2 - k_3|\delta|^2 \tag{47}$$

insuring exponential convergence of $I_3 - \tilde{R}$ to zero.

$$\frac{1}{2}\mathrm{tr}(I_3 - \tilde{R}) = (1 - \cos(\theta)) = 2\sin(\theta)^2$$
$$= ||\pi_a \tilde{R}||_F^2 = \sin(\theta)^2 = \cos(\theta/2)^2 \sin(\theta/2)^2$$

Consequently, it is possible to choose k_2 and k_3 such that the second assumption the lemma is satisfied.

From the expressions of the global trust $T = |TR^d e_3|$ (14) and the control torque Γ (46) we use the relation (6) to obtain the desired speed for each rotor.

4. EXPERIMENTAL RESULTS

In this section, we present experimental results issued from a partial implementation of the above algorithm on the prototype made by the CEA (fig. 1). Note that, as translational velocity measurements are not available only the rotation control is implemented.

4.1 Prototype description

The considered prototype is an X4-flyer equipped with a set of three electronic cards. A first card integrate the controller which regulate the rotation speed the four propellers. The second card integrate an Inertial Measurement Unit constituted of 3 MEMS (low cost sensors) accelerometers and 3 angular rate sensors to provide the components of the gravity in the body frame. On the last card, a Digital Signal Processing (150 MIPS) is embedded and performs all the computations to stabilize the system from the IMU data. A Lithium-Polymer battery allow the power supply during 10 min.

4.2 Results

In our experimentation, the inputs of the considered system are pitch and yaw angles as well as the global thrust T. The algorithm treatment time is 5ms. The evolution of Re_3 during a flight is represented in figure 3. We show that the measured Re_3 converges to the desired vector $R^d e_3$. Moreover, during a stationary flight, the vehicle remains stable and and easy to control by a neophyte.

5. CONCLUSION

In this article, we have proposed a simple nonlinear controller allowing an easy remote control of an unmanned aerial vehicle known as an "X4-flyer". The proposed controller is particulary well adapted for quasi-stationary flight conditions and

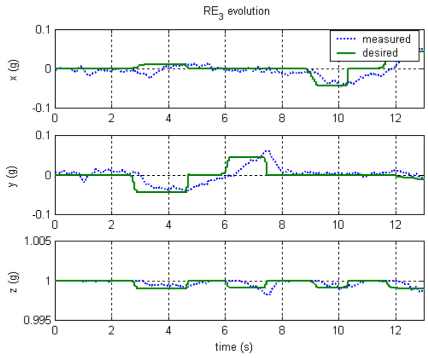

Fig. 3. Re_3 evolution during a flight

assures that the vehicle dynamics remain in the considered flight conditions. The simplicity of the controller allows its implementation on a modest calculator. Experimental results concerning rotational dynamics give good results. Experiments of the full controller will be tested later.

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